OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4733

Probability & Statistics 2

Wednesday **25 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 In a study of urban foxes it is found that on average there are 2 foxes in every 3 acres.

(i) Use a Poisson distribution to find the probability that, at a given moment,

(a) in a randomly chosen area of 3 acres there are at least 4 foxes, [2]

(b) in a randomly chosen area of 1 acre there are exactly 2 foxes. [3]

(ii) Explain briefly why a Poisson distribution might not be a suitable model. [2]

2 The random variable W has the distribution $B(40, \frac{2}{7})$. Use an appropriate approximation to find $P(W > 13)$. [7]

3 The manufacturers of a brand of chocolates claim that, on average, 30% of their chocolates have hard centres. In a random sample of 8 chocolates from this manufacturer, 5 had hard centres. Test, at the 5% significance level, whether there is evidence that the population proportion of chocolates with hard centres is not 30%, stating your hypotheses clearly. Show the values of any relevant probabilities. [7]

4 DVD players are tested after manufacture. The probability that a randomly chosen DVD player is defective is 0.02. The number of defective players in a random sample of size 80 is denoted by R.

(i) Use an appropriate approximation to find $P(R \geqslant 2)$. [4]

(ii) Find the smallest value of r for which $P(R \geqslant r) < 0.01$. [3]

5 In an investment model the increase, Y%, in the value of an investment in one year is modelled as a continuous random variable with the distribution $N(\mu, \frac{1}{4}\mu^2)$. The value of μ depends on the type of investment chosen.

(i) Find $P(Y < 0)$, showing that it is independent of the value of μ. [4]

(ii) Given that $\mu = 6$, find the probability that $Y < 9$ in each of three randomly chosen years. [4]

(iii) Explain why the calculation in part **(ii)** might not be valid if applied to three consecutive years. [1]

6 Alex obtained the actual waist measurements, w inches, of a random sample of 50 pairs of jeans, each of which was labelled as having a 32-inch waist. The results are summarised by

$$n = 50, \quad \Sigma w = 1615.0, \quad \Sigma w^2 = 52\,214.50.$$

Test, at the 0.1% significance level, whether this sample provides evidence that the mean waist measurement of jeans labelled as having 32-inch waists is in fact greater than 32 inches. State your hypotheses clearly. [10]

7 The random variable X has the distribution $N(\mu, 8^2)$. The mean of a random sample of 12 observations of X is denoted by $\overline{X}$. A test is carried out at the 1% significance level of the null hypothesis H_0: $\mu = 80$ against the alternative hypothesis H_1: $\mu < 80$. The test is summarised as follows: 'Reject H_0 if $\overline{X} < c$; otherwise do not reject H_0'.

(i) Calculate the value of c. [4]

(ii) Assuming that $\mu = 80$, state whether the conclusion of the test is correct, results in a Type I error, or results in a Type II error if:

(a) $\overline{X} = 74.0$, [1]

(b) $\overline{X} = 75.0$. [1]

(iii) Independent repetitions of the above test, using the value of c found in part **(i)**, suggest that in fact the probability of rejecting the null hypothesis is 0.06. Use this information to calculate the value of μ. [4]

8 A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where n and k are positive constants.

(i) Find k in terms of n. [3]

(ii) Show that $E(X) = \frac{n+1}{n+2}$. [3]

It is given that $n = 3$.

(iii) Find the variance of X. [3]

(iv) One hundred observations of X are taken, and the mean of the observations is denoted by $\overline{X}$. Write down the approximate distribution of $\overline{X}$, giving the values of any parameters. [3]

(v) Write down the mean and the variance of the random variable Y with probability density function given by

$$g(y) = \begin{cases} 4\left(y + \frac{4}{5}\right)^3 & -\frac{4}{5} \leqslant y \leqslant \frac{1}{5}, \\ 0 & \text{otherwise.} \end{cases}$$ [3]

1	(i) (a) Po(2): 1 – P(≤ 3) = 0.1429	M1 A1	 2	Po(2) tables, "1 – " used Answer, a.r.t. 0.143
	(b) Po(2/3): $e^{-2/3}\frac{(\frac{2}{3})^2}{2!}$ = 0.114	M1 M1 A1	 3	Parameter 2/3 Poisson formula correct, $r = 2$, any μ Answer, a.r.t. 0.114
	(ii) Foxes may congregate so not independent	B1 B1	 2	Independent/not constant rate/singly used Any valid relevant application in context
2	N(80/7, 400/49) $\frac{13.5-\frac{80}{7}}{\frac{20}{7}}$ = 0.725 1 – Φ(0.725) = 0.2343	B1 B1 M1 A1 A1 M1 A1	 7	80/7, a.e.f (11.43) 400/49 or 20/7 seen, a.e.f. (8.163 or 2.857) Standardise with np & npq or $\sqrt{npq}$ or nq, no $\sqrt{n}$ $\sqrt{npq}$ correct 13.5 correct Normal tables used, answer < 0.5 Answer, a.r.t. 0.234 [SR: Binomial, complete expression M1, 0.231 A1 Po(80/7) B1, complete expression M1, 0.260 A1 Normal approx to Poisson, B1B0 M1A0A1 M1A0]
3	H_0: $p = 0.3$ H_1: $p \neq 0.3$ B(8, 0.3) P(≤ 4) = 0.9420; P(> 4) = 0.0580 P(≤ 5) = 0.9887; P(> 5) = 0.0113 Compare 0.025 or critical value 6 Do not reject H_0 Insufficient evidence that manufacturer's claim is wrong	B1 B1 M1 A1 M1 M1 A1√	 7	NH stated, must be this form (or π) AH stated, must be this form (or π) [μ: B1 both] B(8, 0.3) stated or implied Any one of these four probabilities seen *Either* compare P(≥ 5) & 0.025 / P(≤ 4) & 0.975 *Or* critical region ≥ 6 with 5 H_0 not rejected, can be implied, needs essentially correct method Correct conclusion in context [SR: Normal, Poisson: can get B2M1A0M0M1A1 P(≤ 5): first 4 marks. P(= 5): first 3 marks only.]
4	(i) B(80, 0.02) approx Po(1.6) 1 – P(≤ 1) = 1 – 0.5249 = 0.4751	M1 M1 M1 A1	 4	B(80, 0.02) seen or implied, e.g. N(1.6, 1.568) Po(np) used 1 – P(≤ 1) used Answer, a.r.t. 0.475 [SR: Exact: M1 M0 M0, 0.477 A1]
	(ii) P(≤ 4) = 0.9763, P(≥ 5) = 0.0237 P(≤ 5) = 0.9940, P(≥ 6) = 0.0060 Therefore least value is 6	M1 A1 A1	 3	Evidence for correct method, e.g. answer 6 At least one of these probabilities seen Answer 6 only [SR N(1.6,1.568): 2.326 = $(r - 1.6)/\sqrt{1.568}$ M1 r = 5 or (with cc) 6 A1 Exact: M1 A0 A1]

5	(i) $\frac{0-\mu}{\mu/2} = -2$, independent of μ $1 - \Phi(2) = 1 - 0.9772 =$ 0.0228	M1 A1 A1 A1 4	Standardise, allow –, allow $\mu^2/4$ $z = 2$ or -2 z-value independent of μ and any relevant statement Answer, a.r.t. 0.023
	(ii) $\Phi[(9-6)/3]$ $\Phi(1.0) = 0.8413$ $[\Phi(1.0)]^3$ $= 0.59546$	M1 A1 M1 A1 4	Standardise and use Φ *[no √n]* 0.8413 [not 0.1587] Cube previous answer Answer, in range [0.595, 0.596]
	(iii) Annual increases not independent	B1 1	Independence mentioned, in context. Allow "one year affects the next" but not "years not random"
6	H_0: $\mu = 32$; H_1: $\mu > 32$, where μ is population mean waist measurement $\overline{W} = 32.3$ $s^2 = 52214.50/50 - \overline{W}^2$ [= 1] $\hat{\sigma}^2 = 50/49 \times s^2$ [= 50/49 or 1.0204]	B1 B1 B1 M1 M1	One hypothesis correctly stated, *not* x or $\bar{x}$ or $\overline{w}$ Both completely correct, μ used Sample mean 32.3 seen Correct formula for s^2 used Multiply by 50/49 or √
	α: $z = (32.3 - 32)\times\sqrt{49}$ $= 2.1$ Compare 2.1 with 3.09 or 0.0179 with 0.001	M1 A1 B1	Correct formula for z, can use s, aef, need $\mu = 32$ $z = 2.1$ or $1 - \Phi(z) = 0.0179$, *not* –2.1 Explicitly compare their 2.1 with 3.09(0) or their 0.0179 with 0.001
	β: CV $= 32 + 3.09 \div \sqrt{49}$ $= 32.44$ Compare CV with 32.3	M1 B1 A1√	$32 + z \times \sigma/\sqrt{n}$ [allow ±, s, any z] $z = 3.09$ and (later) compare $\bar{x}$ CV in range [32.4, 32.5], √ on k
	Do not reject H_0 Insufficient evidence that waists are actually larger	M1√ A1√ 10	Correct conclusion, can be implied, needs essentially correct method including √n, any reasonable σ, but not from $\mu = 32.3$ Interpreted in context
7	(i) $\frac{80-c}{8/\sqrt{12}} = 2.326$ $c = 74.63$	M1 A1 4 B1 A1	Equate standardised variable to Φ^{-1}, allow – √12, 8 correct 2.326 or a.r.t 2.33 seen, signs must be correct Answer, a.r.t. 74.6, cwo, allow ≤ or ≥
	(ii) (a) Type I error (b) Correct	B1√ 1 B1√ 1	"Type I error" stated, needs evidence "Correct" stated or clearly implied Wrong c: $74 < c < 75$, B1√ B1√ $c < 74$, both "correct", B1. $75 < c < 80$, both "Type I", B1 Also allow if only one is answered
	(iii) $\frac{74.63-\mu}{8/\sqrt{12}} = -1.555$ Solve for μ $\mu = 78.22$	M1*dep A1√ dep*M1 A1 4	$\frac{c-\mu}{8/\sqrt{12}} = (\pm)\Phi^{-1}$, allow no √12 but not 80, not 0.8264 Correct including sign, √ on their c Solve to find μ, dep, answer consistent with signs Answer, a.r.t. 78.2

8	(i)	$\int_0^1 x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$ $k/(n+1) = 1$ so $k = n + 1$	M1 M1 A1 3	Integrate x^n, limits 0 and 1 Equate to 1 and solve for k Answer $n + 1$, *not* 1^{n+1}, c.w.o.
	(ii)	$\int_0^1 x^{n+1} dx = \left[\frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+2}$ $\mu = \frac{k}{n+2} = \frac{n+1}{n+2}$ **AG**	M1 A1 A1 3	Integrate x^{n+1}, limits 0 and 1, not just $x.x^n$ Answer $\frac{1}{n+2}$ Correctly obtain given answer
	(iii)	$\int_0^1 x^5 dx = \left[\frac{x^6}{6}\right]_0^1$ $[= \frac{1}{6}]$ $\sigma^2 = \frac{4}{6} - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$	M1 M1 A1 3	Integrate x^5, limits 0 and 1, allow with n Subtract $\left(\frac{4}{5}\right)^2$ Answer $\frac{2}{75}$ or a.r.t. 0.027
	(iv)	$N\left(\frac{4}{5}, \frac{2}{7500}\right)$	B1 B1 B1√ 3	Normal stated Mean $\frac{4}{5}$ or $\frac{n+1}{n+2}$ Variance their (iii)/100, a.e.f., allow √
	(v)	Same distribution, translated Mean 0 Variance $\frac{2}{75}$	M1 A1√ B1√ 3	Can be negative translation; *or* integration, must include correct method for integral (Their mean) − $\frac{4}{5}$, c.w.d. Variance same as their (iii), or $\frac{2}{75}$ by integration